

CYPRUS MATHEMATICAL SOCIETY



Regional Competition
November 2025

«Lyceum A'»

Date: 8/11/2025

Time: 10:00-12:00

Οδηγίες

1. Solve **all** problems, **justifying** fully your answers.
2. Write using blue or black ink. (Figures can be drawn using a pencil)
3. Correction fluid (Tipp-ex) is not permitted.
4. Calculators are not permitted.
5. Each problem is worth 10 points

Problem 1. Determine the set of values of the parameter $\alpha \in \mathbb{R}$ for which the following equation has solutions in real numbers:

$$\frac{1}{x + \alpha - 1} - \frac{2\alpha}{x^2 - \alpha^2 + 2\alpha - 1} = \frac{5}{x - \alpha + 1}$$

Problem 2.

- i. Fully factorize the following expression into a product of factors

$$A = (x + 2)^4 - x^4$$

- ii. Prove that for every $x > -1$:

$$(2x + 2)^3 < A < (2x + 3)^3$$

- iii. Prove that if ν is a non-negative integers, then $M = \sqrt[3]{(\nu + 2)^4 - \nu^4}$ is not a natural number.

Problem 3. With the notation $\overline{xy\omega}$ we indicate that the number $xy\omega$ is in decimal notation i.e. $\overline{xy\omega} = 100x + 10y + \omega$. Determine all sums of the form

$$S = \overline{4\alpha 5} + \overline{2\beta 7} + \overline{1\gamma 4}$$

which are multiples of 9.

Problem 4. Let $\triangle AB\Gamma$ be a triangle with $AB < A\Gamma$ and M the midpoint of $B\Gamma$. On the side $A\Gamma$, let E be chosen such that $AB = AE$. The perpendicular from the point A to the line BE meets BE at the point K . The parallel from Γ to BE meets AK at the point N . If T is the midpoint of KN prove that:

- i. The lines MT and KN meet perpendicularly.
- ii. The angles $\angle MNT$ and $\angle ABK$ are equal.