Cyprus Mathematical Society

DISTRICT COMPETITION
NOVEMBER 2026

"A′ Lyceum"

Date: 7/11/2026 Time: 10:00-12:00

Instructions

1. Solve **all** the problems, **justifying** your answers fully.
2. Write in blue or black ink. (Figures may be drawn in pencil.)
3. The use of correction fluid is not allowed.
4. The use of calculators is not allowed.
5. Each problem is worth 10 marks.

Problem 1. i. Prove that the numerical expression

$$A = \frac{2+\sqrt{2}}{2-\sqrt{2}} + \frac{2-\sqrt{2}}{2+\sqrt{2}}$$

is a natural number.

ii. For every natural number $\nu > 1$ we consider the number

$$B = \frac{\nu+\sqrt{\nu}}{\nu-\sqrt{\nu}} + \frac{\nu-\sqrt{\nu}}{\nu+\sqrt{\nu}}.$$

Prove that $B = 2 + \dfrac{4}{\nu-1}$ and find the values of ν for which B is a natural number.

Problem 2. i. A natural number is called *strange* if it is one more than a perfect square. Prove that the number 3^4+4 can be written as a product of two strange numbers.
Note: For example, the number $10 = 9+1$ is strange, since $9 = 3^2$ is a perfect square.

ii. Prove that for every integer $\nu \geqslant 2$ the number ν^4+4 is not prime.

iii. Calculate the value of the expression

$$A = \frac{(2^4+4)(6^4+4)(10^4+4)\cdots(2026^4+4)}{(4^4+4)(8^4+4)(12^4+4)\cdots(2028^4+4)}.$$

Problem 3. Let $AB\Gamma\Delta$ be a rectangle with $AB > B\Gamma$. On the side $\Gamma\Delta$ we take a point Z such that $\Gamma Z = B\Gamma$. The perpendicular from the point B to the diagonal $A\Gamma$ meets $A\Gamma$ at the point E. Let O be the point of intersection of the diagonals of the rectangle. The bisector of the angle $\angle OEB$ meets the line BZ at the point K. Prove that:

i. The line BZ is the bisector of the angle $\angle EBO$.

ii. The line OK is parallel to the sides AB and $\Gamma\Delta$ of the rectangle.

Problem 4. i. If p, q are natural numbers, factorise the expression

$$p^2 + pq + 2p + q + 1.$$

ii. If α, β are positive integers and δ is their greatest common divisor, prove that δ divides the difference $\alpha - \beta$.

iii. Find all pairs (p, q) of prime numbers for which the number $p^2 + pq + 2p + q + 1$ is a perfect square.