CYPRUS MATHEMATICAL SOCIETY
B΄ SELECTION COMPETITION
FOR UNDER 15 1/2 YEARS OLD
«Euclidis»

Date: 24/02/2018 **Time duration: 10:00-14:30**

Instructions:

1. Solve all the problems showing your work.
2. Write with blue or black ink. (You may use pencil for figures)
3. Correction fluid (Tipp-ex) is not permitted.
4. Calculators are not permitted.

Problem 1: Consider the set $A_1 = \{\nu, \nu+1, \nu+2\}$, where ν is an odd number such that $\nu < 2016$. We create a sequence $A_1, A_2, \dots, A_i, A_{i+1}, \dots$ of sets, each having three elements, starting with A_1 and making one of the following choices at each step:
1st choice: To obtain A_{i+1}, we choose a positive integer, we add it in two elements of A_i $i = 1,2,\dots$ and leave the third element of A_i the same. *(For example, if we choose the integer $\kappa \in \mathbb{N}$, then A_2 could be equal to $A_2 = \{\nu+\kappa, \nu+1+\kappa, \nu+2\}$.)*
2nd choice: To obtain A_{i+1}, we choose a positive integer, add it in one of the elements of A_i $i = 1,2,\dots$ subtract it from another one of the elements of A_i, and leave the third element of A_i the same. *(For example, if we choose the integer $\mu \in \mathbb{N}$, then A_2 could be equal to $A_2 = \{\nu+\mu, \nu+1, \nu+2-\mu\}$.)*
Decide if it is possible by using this process to end up after some step with the set $A_j = \{2016, 2017, 2018\}$.

Problem 2: Δίνονται τα ψηφία 0,1,2,3,4,5. Determine the sum of all **even** three-digit numbers that can be obtained by using the digits 0,1,2,3,4,5, if repeating the same digit is not allowed.

Problem 3: Let $\beta_i, i = 1,2,3,\dots,2018$ be positive integers such that

$$\frac{1}{\beta_1^3} + \frac{1}{\beta_2^3} + \cdots + \frac{1}{\beta_{2018}^3} = \frac{1}{2}$$

Show that:

α) For every integer $\nu > 1$ it holds that: $\frac{1}{\nu^3} < \frac{1}{2}\left(\frac{1}{\nu-1} - \frac{2}{\nu} + \frac{1}{\nu+1}\right)$

β) At least three of the numbers $\beta_i, i = 1,2,3,\dots,2018$ are equal.

Problem 4: Let $\varDelta AB\varGamma$ be an equilateral triangle and let (c) be a circle with centre A and radius AB. We take a point $\varTheta$ on the arc major $B\varGamma$ of (c) and we draw the chord $B\varTheta$. The parallel line to $B\varTheta$ passing from point $\varGamma$ meets (c) at K. Let $\varDelta, Z, H$ be the midpoints of the segments $B\varTheta, A\varGamma, AK$ respectively. Let Π be a point outside the circle and on the ray $\varDelta A$, and let $\varSigma$ be a point inside the circle such that $\varDelta Z\varPi\varSigma$ is a convex quadrilateral with $\varSigma Z = \varDelta Z$ and $\angle \varDelta\varSigma\varPi = 150°$. Show that:
(α) The triangle $Z\varDelta H$ is equilateral and
(β) $\angle Z\varPi\varDelta = \angle \varDelta\varPi\varSigma$