Cyprus Mathematical Society

DISTRICT COMPETITION
NOVEMBER 2026

"B′ Lyceum"

Date: 7/11/2026 Time: 10:00-12:00

Instructions

1. Solve **all** the problems, **justifying** your answers fully.
2. Write in blue or black ink. (Figures may be drawn in pencil.)
3. The use of correction fluid is not allowed.
4. The use of calculators is not allowed.
5. Each problem is worth 10 marks.

Problem 1. i. A natural number is called *strange* if it is one more than a perfect square. Prove that the number $3^4 + 4$ can be written as a product of two strange numbers.
Note: For example, the number $10 = 9 + 1$ is strange, since $9 = 3^2$ is a perfect square.

ii. Prove that for every integer $\nu \geqslant 2$ the number $\nu^4 + 4$ is not prime.

iii. Calculate the value of the expression

$$A = \frac{(2^4+4)(6^4+4)(10^4+4)\cdots(2026^4+4)}{(4^4+4)(8^4+4)(12^4+4)\cdots(2028^4+4)}.$$

Problem 2. Let $AB\Gamma$ be a triangle.

i. Prove that

$$\cos(A+B)\cdot\cos(A-B) = \frac{1}{2}\left(\cos 2A + \cos 2B\right).$$

ii. If in the triangle $AB\Gamma$, with $\Gamma \neq 90^\circ$, we have

$$\frac{\sin B}{\cos \Gamma} = \sin A + \cos A \cdot \cot B,$$

prove that the triangle $AB\Gamma$ is right-angled.

Problem 3. For every positive integer ν we consider the number

$$A_\nu = 7^{2\nu+1} + 5^{2\nu-1} - 12.$$

i. Prove that A_ν is a multiple of 24 for every positive integer ν.

ii. Find all positive integers ν for which A_ν is a multiple of 48.

Problem 4. Let $AB\Gamma\Delta$ be a parallelogram with acute angle A. Let O be the point of intersection of its diagonals. From the vertex Γ we draw the perpendiculars ΓE and ΓZ to the extensions of the sides AB and $A\Delta$ respectively.

i. Prove that $AB\cdot AE + A\Delta\cdot AZ = A\Gamma^2$.

ii. Prove that the triangle OEZ is isosceles.

iii. Find the value of the angle A for which the triangle OEZ is equilateral.