Күпріакн Мафиматікн Етаіреіа



REGIONAL COMPETITION NOVEMBER 2025 «GYMNASIUM C'»

Date: 08/11/2025 Time: 10:00-12:00

Instructions

1. Solve all problems, justifying fully your answers.

2. Write using blue or black ink. (Figures can be drawn using a pencil)

3. Correction fluid (Tipp-ex) is not permitted.

4. Calculators are not permitted.

Problem 1. Real numbers x, y are given such that x + y = 9 and $x^2 + y^2 = 53$. Calculate the value of the expression

$$A = \frac{1}{x} + \frac{1}{y}.$$

Problem 2.

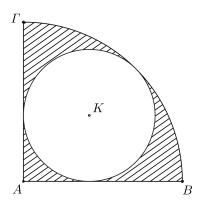
(a) Three real numbers have the following property: the sum of the first two is equal to 3, the sum of the second and third is equal to 11, and the sum of the first and third is equal to 25.

(i) Calculate the sum of the three numbers.

(ii) Find the three numbers.

(b) Three positive real numbers have the following property: the product of the first two is equal to 1, the product of the second and third is equal to 2, and the product of the first and third is equal to 18. Find the three numbers.

Problem 3. In the figure below, a quarter circle $AB\Gamma$ is given, as well as a circle (K,R) that is tangent internally to all three sides of the quarter circle. If the quarter circle has radius $AB=1+\sqrt{2}$, calculate the area of the shaded region.



Problem 4.

(a) Find all two-digit numbers that are equal to 9 times the sum of their digits.

(b) Find all three-digit numbers that are equal to 27 times the sum of their digits.