



Cyprus Mathematical Society

Regional Competition
November 2025

«Lyceum C'»

Date: 8/11/2025

Time: 10:00-12:00

Οδηγίες

1. Solve **all** problems, **justifying** fully your answers.
2. Write using blue or black ink. (Figures can be drawn using a pencil)
3. Correction fluid (Tipp-ex) is not permitted.
4. Calculators are not permitted.
5. Each problem is worth 10 points

Problem 1.

- i. For every positive integer $\nu > 2$, prove that

$$\sqrt{1 + \frac{1}{(\nu-1)^2} + \frac{1}{\nu^2}} = 1 + \frac{1}{\nu-1} - \frac{1}{\nu}$$

- ii. Compute the value of the expression

$$A = \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \sqrt{1 + \frac{1}{3^2} + \frac{1}{4^2}} + \cdots + \sqrt{1 + \frac{1}{99^2} + \frac{1}{100^2}}$$

Problem 2. Let $f : (0, +\infty) \rightarrow \mathbb{R}$ be defined by

$$f(x) = \frac{x^\rho}{2025^x}, \rho \in \mathbb{R}, \rho > 0$$

- i. Investigate the monotonicity and the extrema of the function f .
- ii. Determine, for the various values of $\rho > 0$, the number of positive real roots of the equation

$$x^\rho = 2025^x$$

Problem 3. Determine all positive integers x and y which satisfy the equation

$$\frac{xy}{3} + 2y + x = 2025$$

Problem 4. Let $AB\Gamma\Delta$ be a square of side length α . On the sides $A\Delta$ and $\Gamma\Delta$ we choose points E and Z respectively such that $AE = \Delta Z = \frac{\alpha}{3}$.

Let M and N be the midpoints of EZ and ZB respectively. The line AZ meets the lines MN and EB at the points K and T respectively. If the parallel line through the point B to AZ meets the line MN at the point Λ , prove that the quadrilateral $K\Lambda BT$ είναι ορθογώνιο και να υπολογίσετε το εμβαδόν του συναρτήσει του α .