Cyprus Mathematical Society

DISTRICT COMPETITION
NOVEMBER 2026

"C′ Lyceum"

Date: 7/11/2026 Time: 10:00-12:00

Instructions

1. Solve **all** the problems, **justifying** your answers fully.
2. Write in blue or black ink. (Figures may be drawn in pencil.)
3. The use of correction fluid is not allowed.
4. The use of calculators is not allowed.
5. Each problem is worth 10 marks.

Problem 1. Let $AB\Gamma$ be a triangle.

i. Prove that
$$\cos(A+B)\cdot\cos(A-B)=\frac{1}{2}\left(\cos 2A+\cos 2B\right).$$

ii. If in the triangle $AB\Gamma$, with $\Gamma\neq 90^\circ$, we have
$$\frac{\sin B}{\cos\Gamma}=\sin A+\cos A\cdot\cot B,$$
prove that the triangle $AB\Gamma$ is right-angled.

Problem 2. Consider the function $f:\mathbb{R}\to\mathbb{R}$ with $f(x)=x^2+\alpha x+\beta$, where α,β are real numbers such that
$$\alpha<0,\qquad \beta>0\qquad \text{and}\qquad \beta+1<|\alpha|.$$

i. Prove that the equation $f(x)=0$ has two distinct real roots.

ii. Prove that exactly one of these roots lies between the numbers α and β.

Problem 3. Let $AB\Gamma$ be an acute-angled triangle and let P be a point in the interior of the side $B\Gamma$. The perpendicular bisectors of the segments PB and $P\Gamma$ meet the sides AB and $A\Gamma$ at the points Δ and Z respectively. The bisector of the angle $\angle BA\Gamma$ meets the segment PZ at an interior point K, and the bisector of the angle $\angle\Delta PZ$ meets the segment AZ at an interior point T. Prove that the points A, T, P and K lie on the same circle.

Problem 4. The sequence (α_ν) is defined by
$$\alpha_1=1\qquad\text{and}\qquad \alpha_{\nu+1}=\alpha_\nu+\frac{1}{\alpha_\nu}\quad\text{for every }\nu\geqslant 1.$$

i. Prove that $\alpha_\nu^2\geqslant 2\nu-1$ for every positive integer ν.

ii. Prove that for every integer $\nu\geqslant 2$
$$\alpha_\nu^2\leqslant 2\nu-1+\left(1+\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2\nu-3}\right).$$

iii. Prove that
$$\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\frac{1}{\alpha_3}+\cdots+\frac{1}{\alpha_{1012}}>44.$$