# CYPRUS MATHEMATICAL SOCIETY

## C΄ SELECTION COMPETITION FOR LYCEUM LEVEL

«Ευκλείδης»

**Date: 9/04/2016** **Time duration: 10:00-14:30**

**Instructions:**

1. Solve all the problems showing your work.
2. Write with blue or black ink. (You may use pencil for the figures)
3. Do not use corrector liquid(Tipp-ex).
4. Do not use calculators.

***Problem 1:*** Determine all the triples $(x, y, z)$ of positive integers which satisfy the equation

$$2^x + 3^y = 6^z - 1$$

***Problem 2:*** Find all functions $f: \mathbb{R} \to \mathbb{R}$ such that:

i. $f(-x) = -f(x),\ \ for\ all\ real\ x$

ii. $f(x+1) = f(x) + 1, for\ all\ real\ x$

iii. $f\left(\frac{1}{x}\right) = \frac{f(x)}{x^2}\ \ \ for\ all\ x \in \mathbb{R},\ x \neq 0.$

***Problem 3***: Find the smallest odd integer $\nu$ such that some $\nu - gon$ (not necessarily convex) can be partitioned into parallelograms whose interiors do not overlap.

***Problem 4:*** Quadrilateral $AB\Gamma\Delta$ is inscribed in circle $(\omega)$ with center $O$ , $\angle B = \angle\Delta = 90°$ and $AB = A\Delta < B\Gamma$. Let $X$ point on the segment $B\Delta$. The line $AX$ meets again the $(\omega)$ at $\Sigma$ other than $A$. From the point $X$ we draw the perpendicular to $A\Sigma$ which intersects the arc $A\Delta\Gamma$ at the point $T$. If $M, Z$ the midpoints of the segments $\Sigma T, AO$ respectively , prove that $BZ = ZM$.