Cyprus Mathematical Society

Pancyprian Competition
December 2025

«Lyceum A′»

Date: 13/12/2025 Time: 9:30-12:30

Instructions

1. Solve **all** problems, **justifying** fully your answers.
2. Write using blue or black ink. (Figures can be drawn using a pencil)
3. Correction fluid (Tipp-ex) is not permitted.
4. Calculators are not permitted.
5. Each problem is worth 10 points

Problem 1. With the notation $\overline{xy\omega}$ we denote the three-digit number whose digits, in this order, are x, y, ω. I.e.

$$\overline{xy\omega} = 100x + 10y + \omega.$$

We are given the three-digit number $\overline{\alpha\beta\gamma}$ for which

$$\overline{\beta\gamma\alpha} = (\alpha+\beta+\gamma)^3, \quad \beta \neq 0.$$

Find the value of the expression

$$P = (\alpha+\beta+\gamma)(\overline{\alpha\beta\gamma}+1) + (\alpha+\gamma)^2.$$

Problem 2. Prove that

$$T = \sqrt{994 \cdot 996 \cdot 998 \cdot 1000 + 16}$$

is a natural number.

Problem 3.

(α) Find a polynomial $h(x, y)$ with $x, y \in \mathbb{R}$ such that

$$x^6 + y^6 = (x^2+y^2)h(x, y).$$

(β) Let α, β be real numbers in the interval $\left[0, \frac{\pi}{4}\right]$. Prove that

$$\sin^6\alpha + 3\sin^2\alpha\cos^2\beta + \cos^6\beta = 1$$

if and only if $\alpha = \beta$.

Problem 4. Given an acute triangle $AB\Gamma$ and BE, ΓZ the two altitudes of the triangle from vertices B and Γ respectively. From vertex A we draw a line (ϵ) parallel to $B\Gamma$, and let M be the midpoint of segment $B\Gamma$. The lines ME and MZ intersect the line (ϵ) at points K and Δ respectively. Let I be the second point of intersection of the circumcircles of triangles AKE and $A\Delta Z$. Prove that the triangles IZM and IEM are equal.