# Cyprus Mathematical Society

**Pancyprian Competition**
**December 2025**

## «Lyceum B′»

Date: 13/12/2025 Time: 9:30-12:30

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## Instructions

1. Solve **all** problems, **justifying** fully your answers.
2. Write using blue or black ink. (Figures can be drawn using a pencil)
3. Correction fluid (Tipp-ex) is not permitted.
4. Calculators are not permitted.
5. Each problem is worth 10 points

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**Problem 1.** Let $f$ be a real function such that

$$f\left(\frac{x}{3}\right) = x^2 + x + 1, \quad x \in \mathbb{R}.$$

Find the sum of all values $y$ for which $f(3y) = 7$.

**Problem 2.** If $x, y$ are non-negative real numbers, prove that

$$x^2 + y^2 + 4xy \geqslant 3\sqrt{xy}(x + y).$$

**Problem 3.** Let $m, n$ be real numbers and $a$ a positive integer. The following three numbers are given in increasing order:

$$(a+1)n^2, \quad m^2, \quad a(n+1)^2.$$

Find the greatest possible value of $m^2$.

**Problem 4.** Given an obtuse triangle $AB\Gamma$ with $\angle A\Gamma B > 90^\circ$ and circumcircle $(\omega_1)$ with center $O_1$. Let $(\omega_2)$ be the circumcircle of triangle $AO_1B$ and $O_2$ its center. Suppose that the lines $A\Gamma$, $B\Gamma$, and $O_1O_2$ intersect the circle $(\omega_2)$ again at points $\Delta$, $\Sigma$, and $E$ respectively. If $M$ is the intersection point of the lines $\Delta\Sigma$ and $\Gamma E$, prove that the lines $O_2M$ and $\Delta\Sigma$ are perpendicular.