Κυπριακη Μαθηματικη Εταιρεια

NATIONAL COMPETITION
DECEMBER 2025
«GYMNASIUM C′»

Date: 13/12/2025 Time: 9:30-12:30

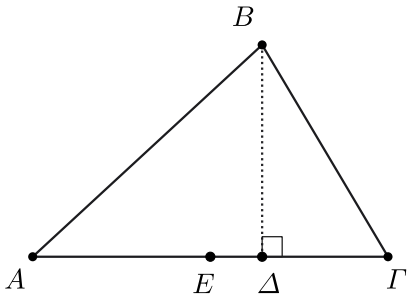
Instructions

1. Solve **all** problems, **justifying** fully your answers.
2. Write using blue or black ink. (Figures can be drawn using a pencil)
3. Correction fluid (Tipp-ex) is not permitted.
4. Calculators are not permitted.

Problem 1. Prove that the sum of any four-digit natural number and its reverse is a multiple of 11.

The reverse of a number is defined as the number obtained by reading its digits in the opposite order. For example, the reverse of 2025 *is* 5202*.*

Problem 2. In the figure below, a triangle $AB\Gamma$ is given, with $AB > B\Gamma$. The midpoint E of the side ΓA is also given, as well as a point Δ on the side ΓA such that $B\Delta$ is an altitude.



(α) Prove that

$$AB^2 = BE^2 + AE^2 + 2AE \cdot E\Delta.$$

(β) Prove that

$$AB^2 + B\Gamma^2 = 2\left(BE^2 + AE^2\right).$$

Problem 3. If $x = \sqrt{2} + \sqrt{3} + \sqrt{5}$, determine integers u, v such that

$$x^2 + \frac{24}{x^2} = u + v\sqrt{6}.$$

Problem 4.

(α) Prove that the number

$$100260169$$

is the square of an integer.

(β) Prove that the number

$$10\ldots0340\ldots0289$$

(where there are exactly 2025 zero digits between 1 and 3, and exactly 2024 zero digits between 4 and 2) is the square of an integer.