SEEMOUS
South Eastern European Mathematical Olympiad for University Students
CYPRUS 2026

Language: **English**

Thursday, March 5, 2026

Problem 1. Determine all differentiable functions $f : \mathbb{R} \to [0, \infty)$ with continuous derivative such that $f^2(x) \leqslant f'(x)$ for all $x \in \mathbb{R}$.

Problem 2. Let A and B be $n \times n$ matrices with complex entries for which

$$A^2 + B^2 = 2(AB - BA).$$

Prove that $A^2 + B^2$ is a nilpotent matrix.

An $n \times n$ matrix M is called *nilpotent*, if there is a positive integer k such that $M^k = O_n$.

Problem 3. Let A and B be $n \times n$ matrices with complex entries for which $A^2 = AB - BA$ and $\operatorname{rank}(A - B) = 1$. Prove that $ABA = O_n$.

Problem 4. Find all bijective functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$x^n f(x) + y^n f(y) \geqslant 2x^n f(y)$$

for all $x, y \in \mathbb{R}$, where n is a fixed positive integer.

Time: 5 hours

Each problem is worth 10 points