Instructions:

1. Solve all the problems. Every problem has 10 points.
2. Write with blue or black ink (you can use pencil for the figures)
3. Use of correction fluid is not allowed.
4. Use of calculators is not allowed.

Problem 1: (α) Find the value of the parameter $\theta \in (0, \pi)$ for which the function f
with $f(x) = \cos^2 x + \cos^2(x + \theta) - \cos x \cdot \cos(x + \theta)$, $x \in \mathbb{R}$ is a constant function.

(b) Find the value of f.

Problem 2: Given an angle $\angle xOy$ and let Oz be its bisector. We take a segment OA on Ox with $(OA) = \alpha$, a segment OB on Oz with $(OB) = \frac{4\alpha}{3}$ and a segment $O\Gamma$ on Oy with $(O\Gamma) = \frac{16\alpha}{9}$. If I is the midpoint of the segment OB and K is the midpoint of the segment $O\Gamma$, prove that the triangles ΔBIA and $\Delta BK\Gamma$ are similar.

Problem 3: We consider a rectangle $AB\Gamma\Delta$ with dimensions α, β and $\alpha \neq \beta$. We draw two parallel lines $(\varepsilon_1), (\varepsilon_2)$ through A, Γ, which have no other common point with the rectangle. We also draw two more lines $(\varepsilon_3), (\varepsilon_4)$ through the points B, Δ that are perpendicular to $(\varepsilon_1), (\varepsilon_2)$. The lines $(\varepsilon_1), (\varepsilon_2), (\varepsilon_3), (\varepsilon_4)$ create a new rectangle $K\Lambda MN$, and let E be its area. Find the maximum value E_{max} of E.

Problem 4: Given the set $A = \left\{2006 + |6^{2\mu} - 5^{\nu}|, \ with \ \mu, \nu \in \{1, 2, 3, \dots\}\right\}$.
Find the minimal element of the set A.