



CYPRUS MATHEMATICAL SOCIETY  
NATIONAL COMPETITION  
DECEMBER 2017

GYMNASIUM C'

Date: 02/12/2017

Time: 09:30 -12:30

**INSTRUCTIONS**

1. Solve all the problems by giving full answers.
2. Each problem is marked with 10 points.
3. Write with blue or black ink (Shapes can be drawn with pencil).
4. The use of corrective liquid (Tip-Ex) is not allowed.
5. The use of a calculator is not allowed.

**PROBLEMS**

**Problem 1**

Let  $x, y$  be positive real numbers, such that the number  $\frac{x}{y}$  is an integer and:

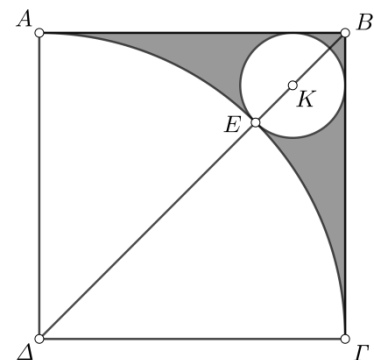
$$\frac{3}{7} < \frac{2x + y}{3x + 10y} < \frac{4}{9}$$

Find the number  $\frac{x}{y}$ .

**Problem 2**

In the adjacent figure  $AB\Gamma\Delta$  is a square with side 1 cm and  $AE\Gamma$  is an arc with center  $\Delta$  and radius  $\Delta\Gamma$ . The circle with center  $K$  and radius  $KE$  is tangent to sides  $AB, B\Gamma$  and arc  $AE\Gamma$ .

- ( $\alpha$ ) Show that  $(KE) = (3 - 2\sqrt{2})$  cm.  
( $\beta$ ) Calculate the shaded area.



**Problem 3**

Calculate the value of the expression:

$$A = \frac{12345^2}{54321 \cdot 66666} + \frac{54321^2}{12345 \cdot 66666} - \frac{66666^2}{12345 \cdot 54321}$$

**Problem 4**

Let  $\alpha, \beta, \gamma$  be integer numbers, such that  $\alpha + \beta + \gamma = 0$ .

Show that the number  $\frac{\alpha^4 + \beta^4 + \gamma^4}{2}$  is a perfect square of an integer.