ΚΥΠΡΙΑΚΗ ΜΑΘΗΜΑΤΙΚΗ ΕΤΑΙΡΕΙΑ
ΠΑΓΚΥΠΡΙΟΣ ΔΙΑΓΩΝΙΣΜΟΣ

Γ΄ ΛΥΚΕΙΟΥ

Ημερομηνία: 2/12/17 *Ώρα εξέτασης: 09:30 -12:30*

Instructions:

1. Solve all the problems. Every problem has 10 points.
2. Write with blue or black ink (you can use pencil for the figures)
3. Use of correction fluid is not allowed.
4. Use of calculators is not allowed.

Problem 1: Given the function : $[0,+\infty) \longrightarrow \mathbb{R}$, with the properties below:

(i) f is continuous on $[0,+\infty)$
(ii) The derivative of f exists on $(0,+\infty)$
(iii) $f(0) = 0$
(iv) f is convex on $(0,+\infty)$.

We define a new function $g:(0,+\infty) \longrightarrow \mathbb{R}$, with $g(x) = \frac{f(x)}{x}$.
Prove that $\pi \cdot f(e) < e \cdot f(\pi)$.

Problem 2: We consider a rectangle $AB\Gamma\Delta$ with dimensions α,β and $\alpha \neq \beta$. We draw two parallel lines $(\varepsilon_1),(\varepsilon_2)$ through A,Γ, which have no other common point with the rectangle. We draw also two more lines $(\varepsilon_3),(\varepsilon_4)$ through the points B,Δ that are perpendicular to $(\varepsilon_1),(\varepsilon_2)$. The lines $(\varepsilon_1),(\varepsilon_2),(\varepsilon_3),(\varepsilon_4)$ create a new rectangle $K\Lambda MN$, and let E be its area. Find the maximum value E_{max} of E.

Problem 3: Given the set $A = \{2006 + |6^{2\mu} - 5^{\nu}|,\ with\ \mu,\nu \in \{1,2,3,\dots\}\}$.
Find the minimal element of the set A.

Problem 4: Given a quadrilateral $AB\Gamma\Delta$, with $\angle B = \angle \Delta = 90^0$. The angle bisectors of $\angle BA\Gamma$ and $\angle B\Gamma A$ meet each other at I and intersect the sides $B\Gamma$ and AB at the points H and Θ, respectively. Let O be the circumcenter of the triangle $\triangle B\Theta H$. The line OI intersects the lines $A\Gamma$ and AB at the points Ξ and N, respectively. Let Z the point of intersection of the diagonals $A\Gamma, B\Delta$ of $AB\Gamma\Delta$ and the circumcircle of the triangle $\triangle A\Delta Z$ intersects again the lines AB and $\Gamma\Delta$ at the points M and K, respectively. If P is the point of intersection of the lines MK and $A\Gamma$, prove that the points M,N,Ξ,P are concyclic.